Eur. Phys. J. C 11, 351–357 (1999)
Digital Object Identifier (DOI) 10.1007/s100529900162

THE EUROPEAN
PHYSICAL JOURNAL C


# Strangeness content and structure function of the nucleon in a statistical quark model

L.A. Trevisan[1], T. Frederico[2], L. Tomio[3]

[1] Departamento de Matemática e Estatística, Universidade Estadual de Ponta Grossa, 84010-790, Ponta Grossa, PR, Brazil (e-mail: latrevis@uepg.br)
[2] Departamento de Física, Instituto Tecnológico de Aeronáutica, CTA, 12228-900, São José dos Campos, SP, Brazil (e-mail: tobias@fis.ita.cta.br)
[3] Instituto de Física Teórica, Universidade Estadual Paulista, 01405-900, São Paulo, SP, Brazil (e-mail: tomio@ift.unesp.br)

Received: 15 March 1999 / Revised version: 15 April 1999 / Published online: 14 October 1999

**Abstract.** The strangeness content of the nucleon is determined from a statistical model using confined quark levels, and is shown to have a good agreement with the corresponding values extracted from experimental data. The quark levels are generated in a Dirac equation that uses a linear confining potential (scalar plus vector). With the requirement that the result for the Gottfried sum rule violation, given by the New Muon Collaboration (NMC), is well reproduced, we also obtain the difference between the structure functions of the proton and neutron, and the corresponding sea quark contributions.

## 1 Introduction

In the last five years, a lot of very precise data on the parton distribution of the nucleon has become available; this data is summarized in [1]. Particularly, the E866 Collaboration [2] has extended the Bjorken-$x$ range of measurement for the asymmetry in Drell–Yan production in pp and pn to $0.03 < x < 0.35$. Such asymmetry gives direct information about the $\bar{\mathrm{u}} - \bar{\mathrm{d}}$ distribution of the sea quark densities. The first indication of sea flavor asymmetry came from the violation of the Gottfried sum rule (GSR) [3] observed by the New Muon Collaboration (NMC) in 1991, which was more precisely determined in 1994 by the same group [4], at $Q^2 = 4$ GeV$^2$. After summing the contributions from the measured and unmeasured regions and adding the errors quadratically, they obtain for the GSR a value given by[1]

$$S_{\mathrm{G}} \equiv \int_0^1 \frac{\mathrm{d}x}{x} \left(F_2^{\mu\mathrm{p}}(x) - F_2^{\mu\mathrm{n}}(x)\right) = 0.235 \pm 0.026. \quad (1)$$

In order to have no violation of the GSR (in case of a symmetric sea) one should obtain a value of 1/3 for the above integral. Recently, the flavor asymmetry was also determined from semi-inclusive deep-inelastic scattering by the HERMES Collaboration [6], in the kinematical region $0.02 < x < 0.3$ and $1 < Q^2 < 10$ GeV$^2$.

Several theoretical models have been proposed for the study of the quark distributions inside the nucleon [5,7], more specifically, the study of the deviation from the GSR. A recent investigation of invariant cross sections for production of $\mathrm{K}^{*-}$ and $\overline{\mathrm{K}^{*0}}$, in the fragmentation region of the proton for p–p and $\gamma$–p reactions also has given a direct and unambiguous probe to the symmetry breaking of the nucleon sea [8], in agreement with NA51 measurements [9]. As shown in [8] the SU(2) asymmetry can be well represented by a simple function given by

$$\frac{\bar{u}(x)}{\bar{d}(x)} = (1-x)^{3.6}. \quad (2)$$

In a quark description for the proton, the Pauli blocking model [10] partially explains the asymmetry as due to the fact that $\mathrm{u}\bar{\mathrm{u}}$ pair creations are more suppressed than $\mathrm{d}\bar{\mathrm{d}}$ (because there are more valence quarks u than d in the proton). The relevance of this effect in the GSR violation has been recently analyzed in [11]. Mesonic models have also been used by several authors [12] to explain the NMC results. Pionic contributions to $\bar{\mathrm{u}} - \bar{\mathrm{d}}$, for example, are considered in [13].

The suggested relevance of the Pauli principle to describe the parton content of the nucleon was considered in the development of quantum statistical models for the parton distribution. In such models, the quarks follow Fermi–Dirac and the gluons Bose–Einstein statistics [15–19]. The approaches used in [15–18] must be distinguished from the one given in [19], despite the common reference to quantum statistics. The statistical distributions assumed

[1] A new global analysis, given in [1], also estimates $S_{\mathrm{G}} \approx 0.27$ at $Q^2 = 4$ GeV$^2$. For a recent review of experimental results and theoretical approaches, see also [5]. In our model, we assume $S_{\mathrm{G}} = 0.24$ for the experimental result. Most of the known experimental data and analyses for the violation of GSR are within a variation of about 30% of the violation given by this value

in [19] are directly given in terms of the momentum fraction $x$ (a continuum variable), with a weight function $f(x)$ that carries the usual parametrization for the parton distribution when the statistical effects are negligible. In the models considered in [15–18] the parametrization in general is done via the single parton interaction in the hadron and by thermodynamical observables, such as the chemical potentials, temperature, and energies. Such "thermal models" enable one to obtain qualitative results at low $Q^2$. The hadrons are described in these models as a confined gas of quarks and gluons at a finite temperature. Mac and Ugaz [16] and Cleymans and Thews [15] have used a Fermi–Dirac distribution with continuum levels for the energies.

Our approach was mainly inspired by the models used in [15] and [16]. However, instead of assuming continuum levels for the quark energies, we consider the Dirac confining potential given in [20] (described in Sect. 2) to generate the single-particle spectrum. In the model, we have a quark gas obeying Fermi statistics. The number of d quarks, which is greater than the number of u quarks in the sea of the proton, at a finite temperature $T$, is parametrized in the model by the different chemical potentials, which are fixed by the normalization of the number of valence quarks inside the hadron. This will result effectively in the Pauli blocking effect. For each flavor $q$ =u, d, and s, we have the corresponding strengths $\lambda_q$ of the confining potential and the current quark masses $m_q$ as parameters, which will be adjusted by the hadron masses. In our model we take $\lambda_{\mathrm{u}} = \lambda_{\mathrm{d}}$ and $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$.

In Sect. 3, we detailed our statistical model, where we assume the GSR violation and the normalizations of the valence quarks inside the nucleon to adjust the main parameters of the model, given by the temperature and chemical potentials. For the antiparticles in the sea to be considered, the chemical potentials for the up and down quarks were introduced to normalize the number of valence quarks in a given sum rule [22]. In such an approach, the nucleon consists of the sea quarks and three valence quarks. The contribution of gluon fields is expected to be small [16,17].

One should observe that the effective potential does not contain all the interactions of the quarks in the hadrons. There are short-range effects, like small-size instanton fluctuations [21] and gluon exchange contributions. The instanton effects give the main contribution to the spin–spin splitting between hadronic multiplets. Thus the pseudoscalar octet loses about one third of its mass as a result of this interaction. In the baryons, the interaction gives a substantial attraction in channels in which there is a scalar diquark, and it determines, in particular, the nucleon (N)–$\Delta$ splitting. Because of such an effect, we use the same $\lambda$ for both $\Delta$ and nucleon. For the s quark, considering the current mass as 150 MeV, the corresponding $\lambda_{\mathrm{s}}$ is adjusted such that the effective mass is about one third of the mass of $\Omega$ at zero temperature. The instanton contribution and temperature effects are further discussed in Sect. 4.

Thus, in our statistical quark model we have not too much freedom, as all the parameters are constrained by the observables: the strengths of the Dirac confining potential and current quark masses, adjusted by hadron masses; the two chemical potentials ($\mu_{\mathrm{u}}$, $\mu_{\mathrm{d}}$), which will be adjusted by the normalization of the nucleon to the number of valence quarks ($u$ and $d$); and the temperature parameter, which will be mainly adjusted by the Gottfried sum rule violation. All these parameters are adjusted in a consistent way, and a reasonable result for the difference of the structure functions of proton and neutron is reached. Another important experimental result, related to the sea-quark distribution inside the nucleon, is the strangeness content. The strangeness content of the nucleon can be measured by two observables given in [14]. It is also important to describe the strangeness described by a quark model, so as to verify the consistency of the model in describing the hadron in terms of its contents. We show that our model gives a consistent result for the corresponding observables, which are not amenable to model parametrization once the GSR violation and the normalizations of the valence quarks inside the nucleon are fixed. As is shown, the two observables related to the strangeness are in good qualitative agreement with the corresponding experimental observables.

The paper is organized as follows. In Sect. 2, we present the Dirac confining potential model and its parametrization. In Sect. 3, a description of our statistical quark model is given. In Sect. 4, we analyze the temperature and instanton effects, in order to obtain our main results, the strangeness and the structure functions. Finally, our main conclusions are summarized in Sect. 5.

## 2 Dirac equation with confining potential

To generate the energy levels of the confined quarks in our statistical model, we use a linear confining (vector plus scalar) potential, given by [20]

$$V(r) = \frac{1}{2}(1+\beta)\lambda r, \tag{3}$$

where $\beta$ is the fourth usual 4×4 Dirac matrix. The strength $\lambda$ of this potential is the same for the quarks u and d, adjusted to obtain the observed mass of the $\Delta(1232)$ isobar at zero temperature. For the strange quark, $\lambda$ is adjusted in the same way to obtain the mass of $\Omega(1676)$. As was briefly explained in the introduction, we adjust $\lambda$ to the $\Delta$ mass instead of the nucleon mass, considering the instanton contributions in the model as given in [21]. The instanton contribution is negative for the nucleon mass and zero for the $\Delta$ mass, suggesting that we should adjust the model parameters to fit the $\Delta(1232)$ isobar and latter consider the instanton contribution to obtain the nucleon mass.

The specific choice of (3) in the coupled Dirac equation leads to an equation similar to the Schrödinger equation, such that one can easily solve it by using the conventional methods of nonrelativistic dynamics. Such a choice also has the advantage that it avoids an old problem that occurs in confining theories when the vector part of the potential is dominant, known as Klein's paradox [23]. This paradox corresponds to a possibility of tunneling even when one has an infinite potential.

The Dirac equation to be solved is given by

$$\left[\vec{\alpha}\cdot\vec{p}+\beta m+\frac{1}{2}(1+\beta)\lambda r\right]\psi_i(\vec{r})=\varepsilon_i\psi_i(\vec{r}), \quad (4)$$

where $\vec{\alpha}$ and $\beta$ are the usual $4\times 4$ Dirac matrices, which can be written in terms of the $2\times 2$ Pauli matrices. With $\psi_i(\vec{r})$ given by

$$\psi_i(\vec{r})=\begin{pmatrix}1\\ \vec{\sigma}\cdot\vec{p}/(m+\varepsilon_i)\end{pmatrix}\varphi_i(\vec{r}), \quad (5)$$

the final coupled equations will be reduced to a single second order differential equation:

$$\left[\vec{p}^2+(m+\varepsilon_i)(m+\lambda r-\varepsilon_i)\right]\varphi_i=0. \quad (6)$$

This equation is solved numerically for the radial part, after partial wave expansion. In case of the s wave ($l=0$), where $j^{\mathrm{p}}=(1/2)^+$, the radial part of $\varphi_i$ is proportional to the Airy function (Ai):

$$\varphi_i(r)=\sqrt{\frac{K_i}{4\pi}}\frac{\mathrm{Ai}(K_i r+a_i)}{r\,[\mathrm{dAi}(x)/\mathrm{d}x]\,|_{x=a_i}}. \quad (7)$$

$a_i$ is the corresponding $i$th root of $\mathrm{Ai}(x)$, $K_i\equiv\sqrt[3]{\lambda(m+\varepsilon_i)}$, $m$ is the current quark mass, and $\varepsilon_i$ the energy levels, which are given by

$$\varepsilon_i=m-\frac{\lambda}{K_i}a_i \quad (8)$$

For the u and d quarks, $m=0$, the energies are given by

$$\varepsilon_i=\sqrt{\lambda}(-a_i)^{\frac{3}{4}}. \quad (9)$$

We have solved (4) for the quarks u, d, and s, using for the current quark masses the values $m_{\mathrm{u}}=m_{\mathrm{d}}=0$ and $m_{\mathrm{s}}=150$ MeV. As explained above, we took the same $\lambda$ for the quarks u and d, such that the ground-state energy of the three-quark system (u and/or d) is equal to the mass of the $\Delta(1232)$ isobar. In the same way, for the s quark the $\lambda$ is adjusted to obtain $\Omega(1676)$. Thus the corresponding ground-state energies are adjusted such that

$$\varepsilon_0\simeq\frac{M_\Delta}{3}\quad\text{and}\quad\varepsilon_0^{(s)}\simeq\frac{M_\Omega}{3}. \quad (10)$$

The numerical values used for $\lambda$, and the corresponding ground-state energies obtained, are given in Table 1.

The particles $\Delta$ and $\Omega$ were considered when the parameters of the potential were chosen, because the instanton contribution is zero for such particles, and is negative for the nucleon [21]. There is no instanton contribution for the spin 3/2 particles, $\Delta(1232)$ and $\Omega(1676)$. Therefore, we have not tried to fix the nucleon mass, but instead we have obtained it by calculating the corresponding instanton effect, using the linear potential given in (3), in the same way as in [21] (through integration of the corresponding Lagrangian density).

**Table 1.** Numerical parameters used for the three quark flavors u, d and s: the strengths of the interaction ($\lambda$), the current masses ($m$), and the corresponding ground-state energies ($\varepsilon$). All the quantities are given in MeV

| | u | d | s |
|---|---|---|---|
| $\lambda$ | 239 | 239 | 312 |
| $m$ | 0 | 0 | 150 |
| $\varepsilon$ | 410 | 410 | 558 |

At zero temperature, the nucleon mass and the instanton contribution ($E_{\mathrm{I}}$) are given by

$$\begin{aligned}M_{\mathrm{N}}(0)=M_\Delta(0)+E_{\mathrm{I}}=&\ (1232-267)\ \mathrm{MeV}\\ =&\ 965\ \mathrm{MeV}.\end{aligned} \quad (11)$$

This will adjust the static properties of the confining potential model. The higher energy levels for the quarks, obtained from the linear potential described above, will contribute to the *thermal masses* of the nucleon as described in the next section. We have determined up to 46 energy levels, and observe that at least the first 30 levels are effective in order to obtain a reasonable accuracy in our model results. Such a number results from the temperature which is necessary to reproduce the GSR violation; we describe this in the next section.

## 3 Statistical quark model

In this section, we describe our statistical quark model and obtain the structure function of the nucleon. The energy levels for the quarks, $\varepsilon_i$, are obtained through the relativistic linear confining quark model [20] presented in the preceding section. In the model, the nucleon consists of three valence quarks and the sea quarks, and neglects the contribution of gluon fields, which is expected to be small [16,17]. In order that antiquarks in the nucleon sea be considered, two nonzero chemical potentials ($\mu_{\mathrm{u}}$ and $\mu_{\mathrm{d}}$) were introduced as parameters to normalize the number of valence quarks u and d [22]. The temperature ($T$) is another parameter in the model, which will be used to adjust the violation of the GSR.

The probability density for a system with energy levels $\varepsilon_i$ and temperature $T$ is given by:

$$\rho_\alpha(\vec{r})=\sum_i g_i\psi_i^\dagger(\vec{r})\psi_i(\vec{r})\frac{1}{1+\exp(\frac{\varepsilon_i-\mu_\alpha}{T})}, \quad (12)$$

where $\alpha$ is the flavor number (=u,d,s,$\bar{\mathrm{u}}$, $\bar{\mathrm{d}}$, $\bar{\mathrm{s}}$), $g_i$ is the level degeneracy, $\mu_\alpha$ $(=-\mu_{\bar{\alpha}})$ is the chemical potential, and $|\psi_i(\vec{r})|^2$ is the density probability for each state, normalized to unity:

$$\int\psi_i^\dagger(\vec{r})\psi_i(\vec{r})\mathrm{d}^3r=1. \quad (13)$$

In the present work we consider only the lighter quarks, u, d, and s. The corresponding current quark masses, as assumed in Sect. 2, are $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$ and $m_{\mathrm{s}} = 150$ MeV. As the strength parameters $\lambda_{\mathrm{u}}$ and $\lambda_{\mathrm{d}}$, in the confining potential model, were also assumed to be equal and different from $\lambda_{\mathrm{s}}$, the energies for the u and d quarks will be the same, differing from the s quarks.

With the above, we obtain the following normalization for the proton:

$$\int [\rho_\alpha(\vec{r}) - \bar{\rho}_\alpha(\vec{r})]\mathrm{d}^3 r = \sum_i g_i \left[\frac{1}{1+\exp(\frac{\varepsilon_i - \mu_\alpha}{T})} - \frac{1}{1+\exp(\frac{\varepsilon_i + \mu_\alpha}{T})}\right] = \begin{cases} 1 & \text{for} \quad \alpha = \mathrm{d} \\ 2 & \text{for} \quad \alpha = \mathrm{u} \end{cases} \quad (14)$$

where $\mu_{\mathrm{u}} \equiv \mu_{\mathrm{u}}^{(\mathrm{proton})}$ and $\mu_{\mathrm{d}} \equiv \mu_{\mathrm{d}}^{(\mathrm{proton})}$. The isospin symmetry implies that, for the neutron we have $\mu_{\mathrm{u}}^{(\mathrm{neutron})} = \mu_{\mathrm{d}}$ and $\mu_{\mathrm{d}}^{(\mathrm{neutron})} = \mu_{\mathrm{u}}$.

The units are such that the Boltzmann constants $k$, $\hbar$, and $c$ are all set to 1. The *thermal mass* of the nucleon, $M_{\mathrm{N}}(T)$, is given in terms of the masses of the valence quarks, $M_{\mathrm{u}}(T)$ and $M_{\mathrm{d}}(T)$:

$$M_{\mathrm{N}}(T) = M_{\mathrm{p}}(T) = M_{\mathrm{n}}(T) = 2M_{\mathrm{u}}(T) + M_{\mathrm{d}}(T), \quad (15)$$

where

$$2M_{\mathrm{u}}(T) \equiv 2M_{\mathrm{u}}^{(\mathrm{proton})}(T) = 2M_{\mathrm{d}}^{(\mathrm{neutron})}(T) = \sum_i g_i \left[\frac{\varepsilon_i}{1+\exp(\frac{\varepsilon_i - \mu_{\mathrm{u}}}{T})} + \frac{\varepsilon_i}{1+\exp(\frac{\varepsilon_i + \mu_{\mathrm{u}}}{T})}\right], \quad (16)$$

$$M_{\mathrm{d}}(T) \equiv M_{\mathrm{d}}^{(\mathrm{proton})}(T) = M_{\mathrm{u}}^{(\mathrm{neutron})}(T) = \sum_i g_i \left[\frac{\varepsilon_i}{1+\exp(\frac{\varepsilon_i - \mu_{\mathrm{d}}}{T})} + \frac{\varepsilon_i}{1+\exp(\frac{\varepsilon_i + \mu_{\mathrm{d}}}{T})}\right]. \quad (17)$$

In order to calculate the structure function for the nucleon, it is convenient to write the wave function in momentum space by taking the Fourier transform:

$$\varPhi_i(\vec{p}) = \frac{1}{(2\pi)^{3/2}} \int \exp(-\mathrm{i}\vec{p}\cdot\vec{r})\psi_i(\vec{r})\mathrm{d}^3 r. \quad (18)$$

Using the null plane variables,

$$p^+ = xP^+ , \quad P^+ = M_{\mathrm{N}}(T) , \quad p_z = p^+ - \varepsilon_i = M_{\mathrm{N}}(T)\left(x - \frac{\varepsilon_i}{M_{\mathrm{N}}(T)}\right), \quad (19)$$

where $x$ is the momentum fraction of the nucleon carried by the quark, we can redefine the above wave function as

$$\varPhi_i(\vec{p}) \equiv \varPhi_i(x, \vec{p}_\perp). \quad (20)$$

By using the above (12)–(20) for each quark flavor $\alpha$, we obtain the following structure function:

$$f_\alpha(x) = \sum_i \int \left|\varPhi_i\left(M_{\mathrm{N}}(T)\left(x - \frac{\varepsilon_i}{M_{\mathrm{N}}(T)}\right), \vec{p}_\perp\right)\right|^2 \times \frac{\mathrm{d}^2 p_\perp}{1+\exp\left(\frac{\varepsilon_i - \mu_\alpha}{T}\right)}, \quad (21)$$

which describes the probability that a quark with flavor $\alpha$ has a fraction $x$ of the total momentum of the nucleon. The nucleon structure function, $F_2^{\mathrm{N}}(x)$, is given by

$$F_2^{\mathrm{N}}(x) = \sum_\alpha e_\alpha^2 x f_\alpha(x), \quad (22)$$

where $e_\alpha$ is the electric charge of the corresponding quark flavor. For notation convenience, we replace $f_\alpha(x)$ by $\alpha(x)$ in the following.

The chemical potentials $\mu_{\mathrm{u}}$ and $\mu_{\mathrm{d}}$ are used to normalize the number of valence quarks of the nucleon, such that for the proton we have, instead of (14),

$$\int_0^1 [u(x) - \bar{u}(x)]\,\mathrm{d}x = 2 , \quad \int_0^1 \left[d(x) - \bar{d}(x)\right]\mathrm{d}x = 1 \quad (23)$$

$$\int_0^1 [s(x) - \bar{s}(x)]\,\mathrm{d}x = 0 . \quad (24)$$

For the neutron, we should exchange the flavors u and d in (23). $\mu_{\mathrm{u}}$, $\mu_{\mathrm{d}}$, and $T$ are adjusted with three observables, respectively given by the normalizations of the valence quarks u and d [see (23)] and by the GSR [see (1)]. The chemical potential corresponding to the strange quark is zero in the nucleon case, as given by (24).

The temperature parameter $T$ is determined by consideration of the violation of the Gottfried sum rule [3], which can be obtained from (1) [4]. Assuming $S_{\mathrm{G}} = 0.24$ as the experimental result, the "experimental" violation of the GSR is given by

$$I_{\mathrm{GSR}} \equiv \frac{3}{2}S_{\mathrm{G}} - \frac{1}{2} = \int_0^1 \left(\bar{u}(x) - \bar{d}(x)\right)\mathrm{d}x = -0.14 . \quad (25)$$

The strengths of the confining potential in the Dirac equation, for the quarks u, d, and s, are fixed at $T = 0$ through baryon masses which have zero instanton contribution; we describe this in the following section.

## 4 Temperature effect and strangeness

Since we are working in a statistical thermal model, observables such as the particle masses will depend on a temperature parameter, given in (15)–(17) by the *thermal masses*. Other observables, e.g., the chemical potentials and the Gottfried sum rule, will also depend on such a temperature parameter. We consider such dependence in the following.

**Table 2.** The chemical potentials and thermal masses at different temperatures, without instanton effects. All the quantities are given in MeV

| $T$ | $\mu_u$ | $\mu_d$ | $M_u$ | $M_d$ | $M_\Delta$ |
|---|---|---|---|---|---|
| 100 | 185 | 121 | 622.5 | 620 | 1866 |
| 110 | 145 | 85 | 657 | 650 | 1964 |
| 120 | 109 | 60 | 686 | 691 | 2063 |

Initially, in our first parametrization of the model, we have not considered short-range effects, such as the small-size instanton fluctuations [21], which can be responsible for the N–$\Delta$ mass difference. In this way, the ground-state energies for the quarks are adjusted to reproduce the Delta isobar mass (a particle with no instanton contribution) at $T = 0$, such that $\varepsilon_0 = 410$ MeV. In Table 2, we present the corresponding results for the chemical potentials and the thermal masses in MeV. We observe that the chemical potential decreases as the temperature and thermal masses increase. In order to reproduce the GSR violation, we need to increase the temperature. As we increase the temperature, we excite more states from the vacuum; this explains the large thermal masses we have[2]. The chemical potential decreases in order to keep fixed the corresponding normalization, given by (23) and (24).

## 4.1 Instanton effect

Next, observing that the effective potential does not contain all the interactions of the quarks in the hadrons, we consider the contribution of small-size instanton fluctuations [21]. These interactions can be considered as the main reason for the spin–spin splitting between hadronic multiplets, such that the pseudoscalar octet loses about one third of its mass as a result of this interaction. There are also other contributions to the N–$\Delta$ splitting that we have not included in the present model, e.g., the gluon exchange corrections. As is shown in [21], the contribution for the splitting coming from gluon exchange is of the order of 40 MeV. Therefore, the instanton contribution determines the main part of the N–$\Delta$ splitting in our approach. When only the instanton contribution to the splitting is considered, the mass of the nucleon becomes higher than the observed one. However, for the purpose of the present preliminary approach of the statistical quark model, such a difference is not so relevant. We need to keep in mind that the addition of gluon effects cannot be avoided when one tries to reproduce the behavior of the structure functions at very low $x$. The addition of such effects, in a model that is an extension of the present one, is being considered for a future work.

As the total instanton contribution to the nucleon mass is $-267$ MeV at $T = 0$ [see (11)], the effective ground-state energy of the quarks in the nucleon will be reduced

[2] In this procedure, for a temperature of the order of 110 MeV, we have considered up to 46 levels of the energy spectrum, where at least 30 levels are effective to obtain a precision of two digits in the GSR

**Table 3.** The chemical potentials and thermal masses at different temperatures, with instanton effects in the first energy level taken into consideration. All quantities are given in MeV

| $T$ | $\mu_u$ | $\mu_d$ | $M_u$ | $M_d$ | $M_N$ |
|---|---|---|---|---|---|
| 100 | 157 | 96 | 526 | 521 | 1574 |
| 110 | 125 | 70 | 576 | 568 | 1722 |
| 120 | 71 | 40 | 620 | 615 | 1857 |

**Table 4.** The violation of the Gottfried sum rule, given by $I_{GSR}$ (25), with instanton contributions to the first energy level taken into consideration

| $T$(MeV) | 100 | 110 | 120 |
|---|---|---|---|
| $I_{GSR}$ | $-0.0827$ | $-0.1586$ | $-0.2394$ |

to $\varepsilon_0 = 322$ MeV. This state is predominant in the sums given in (16) and (17), implying that the thermal masses obtained in Table 2 must decrease correspondingly, as is shown in Table 3.

By using the results shown in Tables 2 and 3 we can adjust, in a consistent calculation, the temperature to obtain the corresponding violation of the GSR, given in (25).

In Table 4, we verify how much the temperature affects the results obtained for the violation of the GSR, in which instanton contributions are included in the first energy level. We can observe that the temperature is near to 110 MeV. In fact, to obtain $I_{GSR} = -0.14$, we found that the best choice for the temperature is 108 MeV.

In Fig. 1, we illustrate the model for the difference between the structure functions of the proton and neutron, and we also include the corresponding absolute value of the contribution coming from the sea quarks (dashed line). Such a contribution is given by $2/3x(\bar{\mathrm{d}} - \bar{\mathrm{u}})$ . We adjusted the temperature ($T$ =108 MeV) using (1); and obtained the chemical potentials ($\mu_u$ =135 MeV and $\mu_d$ =78 MeV) by fitting (14). The results of our model are compared with the experimental available ones. One can observe that the model gives a reasonable fitting for such difference at $x$ higher than 0.4. The addition of gluon effects will be relevant for lower $x$.

## 4.2 Strangeness

For the strange-quark content of the nucleon, the relevant observables are given by [22]

$$\eta = \frac{2\int_0^1 xs(x)\mathrm{d}x}{\int_0^1 x\left(u(x) + d(x)\right)\mathrm{d}x},$$
$$\kappa = \frac{2\int_0^1 xs(x)\mathrm{d}x}{\int_0^1 x\left(\bar{u}(x) + \bar{d}(x)\right)\mathrm{d}x}. \tag{26}$$

$\kappa$ represents the ratio between the strange and nonstrange quarks of the sea. With $\eta/\kappa$ we obtain the ratio between the nonstrange antiquarks and quarks in the nucleon:

$$\frac{\eta}{\kappa} = \frac{\int_0^1 \left(\bar{u}(x) + \bar{d}(x)\right) x\mathrm{d}x}{\int_0^1 \left(u(x) + d(x)\right) x\mathrm{d}x}. \tag{27}$$

**Table 5.** The strangeness of the nucleon, given by $\kappa$, $\eta$ and the ratio $\kappa/\eta$, are shown, after adjustment of the violation of the GSR with the temperature used as a parameter. The first two lines gives the best model fitting, for when the instanton contribution is absent and when it is included. In the third line $\Delta$I is the instanton contribution to the second one. For the strangeness observables, the experimental values were obtained from [14]. The quoted experimental result for $I_{\rm GSR}$ corresponds to $S_{\rm G} = 0.24$, which is close to the central value of the NMC data (1994) [4], given in (1)

| | $T$(MeV) | $I_{\rm GSR}$ | $\eta/\kappa$ | $\eta$ | $\kappa$ |
|---|---|---|---|---|---|
| No instanton | 113 | −0.1394 | 0.1580 | 0.1160 | 0.7280 |
| With instanton | 108 | −0.1404 | 0.1570 | 0.0847 | 0.5360 |
| $\Delta$I | 108 | −0.0421 | 0.0505 | −0.0043 | −0.2976 |
| Experimental | – | −0.14 | 0.2075 | $0.099^{+0.009}_{-0.006}$ | $0.477^{+0.063}_{-0.053}$ |

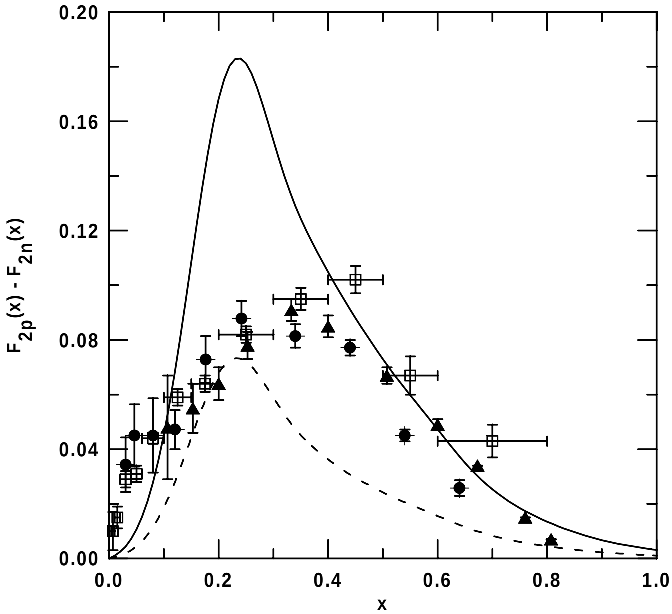


**Fig. 1.** The difference of the structure functions for the proton and neutron, $F_2^{\rm p}(x)-F_2^{\rm n}(x)$, obtained by the model (solid line), is compared with experimental results. Empty squares: from NMC (1994) results [4] at $Q^2 = 4$ GeV$^2$; triangles: from [24], in the range of $7 < Q^2 < 170$ GeV$^2$; circles: from SLAC [25] for $2 < Q^2 < 20$ GeV$^2$; dashed line: the absolute value of the antiquark contribution to this difference, given by $2/3(\bar{\rm d} - \bar{\rm u})$

In Table 5, we present our numerical values for the strangeness given by (26) and (27), with the corresponding values for the temperature parameter and the violation of the GSR. The experimental data for the strangeness were obtained from [14].

For $I_{\rm GSR}$, we consider the value given in (25) as the experimental one. It was used to adjust approximately the corresponding temperatures in our calculations. In our calculations, as explained before, we have also adjusted the chemical potentials to the rules given in (23) and (24). In the line labeled "No instanton", the best fitting was done without consideration of instanton effects. In the line labeled "With instanton", the corresponding fitting was done after consideration of the instanton effect. A comparison with the results given in Table 4 shows that one can also adjust the temperature to a value near 110 MeV. In the third line, we estimate the instanton contribution ($\Delta$I) to the values obtained in the second line. Without instanton contribution, we have already a reasonably good result for the strangeness, as is shown in the first line of Table 5. The results for $\eta$ and $\kappa$ were reduced when the instanton contribution was considered (second line); this improved the agreement with the given experimental values.

## 5 Conclusions

We have presented a thermal model with confining potential, following a similar relativistic quark gas model to that developed in [16]. In our approach, to generate the quark levels, we used the scalar plus vector linear potential given in (3), in which two strength parameters are used, in order to fix the ground state of the particles $\Delta$ and $\Omega$. Considering the current quark masses to be equal to zero for the $u$ and $d$ quarks and 150 MeV for the quark $s$, we obtained effective thermal masses for the constituent quarks and for the hadrons by using a Fermi–Dirac distribution. The thermal mass for the hadron was defined by a sum over the eigenstates of energy, as given in (15), such that the observed baryon mass is reached in the limit $T \to 0$. The effective temperature in the model was fixed to reproduce the violation of the Gottfried sum rule after adjustment of the chemical potentials for u and d quarks. Such chemical potentials were adjusted by normalization of the number of valence quarks in the proton and neutron. For the temperature, we obtained $T = 108$ MeV when we considered the instanton effects to the first energy level, as has been described in the previous sections. For the chemical potentials, the values obtained were $\mu_{\rm u} \approx 135$ MeV and $\mu_{\rm d} \approx 78$ MeV. These results for the temperature and chemical potentials are consistent with the ones obtained in [17] ($T = 103$ MeV, $\mu_{\rm u} = 148$ Mev, $\mu_{\rm d} = 83.4$ MeV). The small differences obtained between the two approaches are due to the corresponding confining quark models which were used, and to the addition of instanton effects in the present approach.

One of the advantages of this approach is that the Pauli blocking effect, suggested to partially explain the asymmetry in the nucleon sea, is naturally incorporated inside the model, with the quarks obeying Fermi–Dirac statistics.

The main new result obtained by the present quark model was the strangeness content of the nucleon, represented by the observables $\eta$ and $\kappa$, which are defined in (26) and (27), as shown in Table 5. In the present work we made the assumption that the strange quark, inside the nucleon, feels the same average central potential as in the $\Omega(1676)$ particle. Considering the results obtained by the model for the observables $\eta$ and $\kappa$, this naive idea looks to be a good approach. We should note that the experimental numbers for the strangeness quoted in Table 5 were extracted at relatively large $Q^2$, and that in the present stage, the model ignores QCD effects. In view of the fact that qualitatively the model gives a good description of such experimental data, one can interpret our results as an evidence that gluonic effects and $Q^2$ dependence are mostly canceled when one takes the ratio of the quark structure functions, as given by (26) and (27). In other words, the high $Q^2$ effect is roughly flavor-independent, and the main effect in the quark structure functions can be factorized. By taking into account $Q^2$ dependence by evolution, and having a better description of the quark structure functions, one can verify the range of validity of the above factorization hypothesis.

Another result obtained from the model is given by the differences of the structure functions for the proton and neutron, which are shown in Fig. 1. As is expected in this kind of simplified quark model, the agreement of the model is better for higher $x$ when compared with the available experimental data. The corresponding contribution of the asymmetry in the sea $(\bar{\mathrm{d}} - \bar{\mathrm{u}})$ is also presented in Fig. 1. In a preliminary approach [26], it was observed that the addition of gluon effects improves the agreement of the results for lower values of $x$, particularly for the ratio between the structure functions of proton and neutron. Because in the present paper we are using a model where the quarks are bound, one should not expect a close agreement of the results for the structure functions; this point has already been discussed in [27] and [17]: All bound states dominated by a three-quark configuration when approaching a DIS regime produce a peak near $x = 1/3$. However, considering the physical appeal of the model, which incorporates the Pauli principle in a natural way, and considering we have no free parameters to obtain the results (for the strangeness content and the differences of the structure functions), we can suggest it as a starting model in the development of a more realistic description. By performing the evolution to high $Q^2$, we believe the present results will be improved, as new parameters will enter in order to describe the structure of the constituent quark (convolution) and gluonic effects.

*Acknowledgements.* We thank Prof. A. Dorokhov, Prof. J. Dey, and Dr. A. Gammal for useful discussions. We also would like to thank the referee of this paper for relevant comments concerning the presentation of our results. This work was partially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

## References

1. A.D. Martin, R.G. Roberts, W.J. Stirling, and R.S. Thorne, Eur. Phys. J. C **4**, 463 (1998)
2. E866 Collaboration: E.A. Hawker, et al., Phys. Rev. Lett. **80**, 3715 (1998)
3. K. Gottfried, Phys. Rev. Lett. **18**, 1174 (1967)
4. NMC Collaboration: P. Amaudruz, et al., Phys. Rev. Lett. **66**, 2712 (1991); M. Arneodo, et al., Phys. Rev. D **50**, R1 (1994)
5. S. Kumano, Phys. Rep. **303**, 183 (1998)
6. HERMES Collaboration: K. Ackerstaff, et al., Phys. Rev. Lett. **81**, 5519 (1998)
7. K.F. Liu and S.J. Dong, Phys. Rev. Lett. **72**, 1790 (1994); M. Anselmino, V. Barone, F. Caruso, and E. Predazzi, Z. Phys. C **55**, 97 (1992); M. I. Pvković, Phys. Rev. **D13**, 2128 (1976)
8. F. Arash and L. Tomio, Phys. Lett. B **401**, 207 (1997)
9. NA51 Collaboration, A. Baldit, et al., Phys. Lett. B **332**, 244 (1994)
10. R.D. Field and R.P. Feynman, Phys. Rev. D **15**, 2590 1977); J.F. Donoghue and E. Golowich, Phys. Rev. D **15**, 3421 (1977)
11. F.M. Steffens and A.W. Thomas, Phys. Rev. C **55**, 900 (1997)
12. E.M. Henley and G.A. Miller, Phys. Lett. B **251**, 453 (1990); S. Kumano and J.T. Londergan, Phys. Rev. D **44**, 717 (1991); A.W. Schreider, P.J. Mulders, A.I. Signal, and A.W. Thomas, Phys. Rev. D **45**, 3069 (1992); W.-Y.P. Hwang, J. Speth, and G.E. Brown, Z. Phys. A **339**, 383 (1991); B.C. Pearce, J. Speth, and A. Szczurek, Phys. Rep. **242**, 193 (1994)
13. C.M. Shakin and W.-D. Sun, Phys. Rev. C **50**, 2553 (1994)
14. A.O. Bazarko, et al., Z. Phys. C **65**, 189 (1995)
15. J. Cleymans and R.L. Thews, Z. Phys. C **37**, 315 (1988)
16. E. Mac and E. Ugaz, Z. Phys. C **43**, 655 (1989)
17. J. Dey, L. Tomio, and M. Dey, Phys. Lett. B **318**, 387 (1993); Phys. Lett. B **320**, 431 (1994)
18. R.P. Bickerstaff and J.T. Londergan, Phys. Rev. D **42**, 3621 (1990); K. Ganesamurthy, V. Devanathan, M. Rajasekaran, and Z. Phys. **C52**, 589 (1991)
19. C. Bourrely, F. Buccella, G. Miele, G. Migliore, J. Soffer, and V. Tibullo, Z. Phys. C **62**, 431 (1994); F. Buccella, G. Miele, and N. Tancredi, Progr. Th. Phys. **96**, 749 (1996); F. Buccella, I. Dorsner, O. Pisanti, L. Rosa, and P. Santorelli, Mod. Phys. Lett. A **13**, 441 (1998)
20. P.L. Ferreira, J.A. Helayel,and N. Zagury, Nuovo Cim. A **55**, 215 (1980)
21. A.E. Dorokhov, N.I. Kochelev, Yu.A. Zubov, Sov. J. of Part. Nucl. **23**, 522 (1992)
22. R.G. Roberts, *The Structure of the Proton – Deep Inelastic Scattering* (Cambridge University Press, Cambridge 1990)
23. O.Klein, Z. Phys. **53**, 157 (1929)
24. J.J. Aubert, et al., Nucl. Phys. B **293**, 740 (1987)
25. S. Stein, et al., Phys. Rev. **12**, 1884 (1975); A. Bodek, et al., Phys. Rev. Lett. **30**, 1087 (1973)
26. L.A. Trevisan, *Thermal model with confining potential for the nucleon structure function*, Ph.D. thesis, Universidade de São Paulo, São Paulo, Brazil, 1997 (in Portuguese)
27. E. Reya, "The Spin Structure of the Nucleon", plenary talk presented at the Workshop on QCD – 20 Years Later (World Scientific, Singapore, 1993)